



Thermally driven escape from Pluto's atmosphere: A combined fluid/kinetic model

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ABSTRACT

A combined fluid/kinetic model is developed to calculate thermally driven escape of N_2 from Pluto's atmosphere for two solar heating conditions: no heating above 1450 km and solar minimum heating conditions. In the combined model, one-dimensional fluid equations are applied for the dense part of the atmosphere, while the exobase region is described by a kinetic model and calculated by the direct simulation Monte Carlo method. Fluid and kinetic parts of the model are iteratively solved in order to maintain constant total mass and energy fluxes through the simulation region. Although the atmosphere was found to be highly extended, with an exobase altitude at ~ 6000 km at solar minimum, the outflow remained subsonic and the escape rate was within a factor of two of the Jeans rate for the exobase temperatures determined. This picture is drastically different from recent predictions obtained solely using a fluid model which, in itself, requires assumptions about atmospheric density, flow velocity and energy flux carried away by escaping molecules at infinity. Gas temperature, density, velocity and heat flux versus radial distance are consistent between the hydrodynamic and kinetic model up to the exobase, only when the energy flux across the lower boundary and escape rate used to solve the hydrodynamic equations is obtained from the kinetic model. This limits the applicability of fluid models to atmospheric escape problems. Finally, the recent discovery of CO at high altitudes, the effect of Charon and the conditions at the New Horizon encounter are briefly considered.

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1. Introduction

Our understanding of Pluto's N_2 dominated atmosphere is largely based on rare occultation observations in 1988 (pre-perihelion), 2002 and 2006 (post-perihelion) (Elliot et al., 2007; Young et al., 2008). Each observation determined the surface pressure to be between 6.5 and 24 μ bars with a peak atmospheric temperature of ~ 100 K at a radial distance ~ 1250 km. At this temperature and due to Pluto's low gravitational energy, e.g. 0.007 eV/amu at 1250 km, escape can result in significant atmospheric loss. Therefore, many studies of Pluto's evolution have been aimed at understanding the loss rate over time (Hubbard et al., 1990; Hunten and Watson, 1982; Strobel, 2008a; Tian and Toon, 2005; Trafton, 1980).

Using results from the occultation observations and calculations of solar heating rates in continuum models of the upper atmosphere, a series of authors calculated thermal escape rates from Pluto's atmosphere by considering a process referred to as slow hydrodynamic escape (SHE, e.g., McNutt, 1989; Krasnopolsky, 1999; Strobel, 2008a). The SHE model of the atmosphere is based on the premise that thermal energy of molecules is efficiently

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converted into bulk flow energy. This assumption can lead to an over-estimate of the escape rate (Johnson, 2010; Volkov et al., 2011a,b) when applied to the rarefied region of the atmosphere where the collisions are too infrequent to maintain local thermal equilibrium. Using a combined fluid/kinetic model to directly account for the non-equilibrium nature of the gas flow in the upper atmosphere, thermally driven escape from Pluto's atmosphere is found to occur on a molecule by molecule basis resulting in an enhanced Jeans escape rate (Tucker et al., 2011).

Globally averaged escape rates typically have been estimated using fluid (Strobel, 2008a,b) or kinetic models (Tucker and Johnson, 2009; Volkov et al., 2011a,b). Kinetic models can in principle be applied to the entire atmosphere, but they are computationally expensive when applied to a dense region of the atmosphere. In the dense region of the atmosphere where collisions are frequent the flow can be treated as a continuum and the use of a fluid model is most efficient. In the fluid models the hydrodynamic equations are solved to obtain the mass flow rate through the atmosphere but are unable to account for relatively infrequent collisions of upwardly moving molecules or returning molecules that regulate escape in the upper atmosphere. Therefore, they cannot correctly calculate the amount of heat transported through the atmosphere. A combined fluid/kinetic model applied to dense and rarefied parts of the atmosphere respectively avoids these difficulties and provides a computationally-efficient tool for simulation of the atmosphere (e.g. Marconi et al., 1996).

The aim of the present paper is to obtain the globally averaged escape rate, gas density, temperature, velocity, and the heat flux in Pluto's atmosphere in the region between 1450 km and 30,000 km using a combined fluid/kinetic model. We numerically solve the one-dimensional (1D) hydrodynamic equations coupled to a molecular kinetic model for the rarefied region of the atmosphere similar to the approach used in Marconi et al. (1996). Preliminary results were given in Tucker et al. (2011). Here simulations are performed for no heating and solar minimum heating conditions. The interaction of Pluto's extended atmosphere with Charon and the implications of the recently discovered CO detection in Pluto's extended atmosphere (Greaves et al., 2011) are briefly considered. The results presented here also suggest that the application of the hydrodynamic models to escape from other planetary atmospheres (e.g., Murray-Clay et al., 2009; Tian, 2009; Strobel, 2008a,b) can give incorrect estimates of the macroscopic properties and the escape flux.

2. Jeans, hydrodynamic, and slow hydrodynamic escape

Although escape driven by solar heating is by its nature a three-dimensional (3D) process, for comparison with previous models, the thermal escape problem is formulated here by considering a 1D, globally averaged, steady-state model of the atmosphere, where the gas properties are functions of the radial distance r , from the planet center. In this section, the applicability of the fluid model to thermal escape is briefly analyzed. Based on this analysis, a combined fluid/kinetic model is introduced in the next section.

The Jeans parameter, $\lambda(r)$, the ratio of the gravitational energy of a molecule $\Phi_g = GM_p m/r$ to its thermal energy kT , is often used to characterize the atmospheric escape rate: i.e., $\lambda(r) = \Phi_g/kT$, with G the gravitational constant, M_p the planet mass, m the molecular mass, k the Boltzmann constant and T the temperature at r . In order to escape a planet's gravity a molecule must be directed outward from the planet, have a velocity larger than $v_{\text{esc}} = \sqrt{2GM_p/r}$ and have a low probability of colliding with other molecules along its trajectory. The density of planetary atmospheres decreases exponentially with altitude and the most rarefied region is referred to as the exosphere. In this region intermolecular collisions are rare, therefore the Jeans parameter is typically evaluated at the lower boundary of the exosphere which is referred to as the exobase r_x . Throughout the paper the subscript "x" will be used to denote the values of all parameters evaluated at $r = r_x$.

The degree of rarefaction of a gas is determined by the local Knudsen number, $Kn = l_c/l_a$, the ratio of the mean free path of the molecules, $l_c \approx c/(n\sigma)$, to an appropriate length scale for the gas density l_a . Here σ is the molecular cross section, n is the local number density, and the numerical coefficient c depends on the model of intermolecular collisions, e. g., for hard sphere molecules $c = 1/\sqrt{2}$ (Bird, 1994; Chapman and Cowling, 1970). The appropriate length l_a for planetary atmospheres is usually defined as the distance over which the density decreases by a factor of $1/e$ and is called the atmospheric scale height $H = r/\lambda(r)$, so that $Kn = l_c/H$. When $Kn \ll 0.1$ the atmosphere is relatively dense so that molecules collide frequently. With increasing altitude collisions become less frequent and the exobase altitude is defined to occur where $Kn_x \sim 1$ or $n_x H_x \sigma \sim 1$.

Three regimes of escape are typically characterized using the Jeans parameter. If a planetary atmosphere has a relatively large Jeans parameter at the exobase for the dominant atmospheric species, thermal escape occurs on a molecule by molecule basis similar to evaporation, a process referred to as Jeans escape (e.g., Chamberlain and Hunten, 1987; Jeans, 1916). In this approximation it is assumed the speed distribution at the exobase is Maxwellian, so that the molecular loss rate is $\phi_j = \pi r_x^2 n_x \langle v_{\text{th},x} \rangle (1 + \lambda_x) \exp(-\lambda_x)$, where $\langle v_{\text{th},x} \rangle = (8kT_x/\pi m)^{1/2}$ is the mean thermal speed of molecules at the

exobase. The concomitant cooling rate, total energy of molecules escaping the atmosphere per unit time, is $\langle E\phi \rangle_j = (kT_x)(2 + 1/(1 + \lambda_x))\phi_j$. Modified Jeans escape rates, accounting for the non-zero gas velocity at the exobase, have also been proposed (e. g. Chamberlain, 1961; Yelle, 2004; Volkov et al., 2011a,b).

At small Jeans parameters the thermal energy of molecules is comparable to or larger than the gravitational binding energy of the planet at the exobase, so the bulk atmosphere can escape as a hydrodynamic outflow (e.g., Öpik, 1963; Hunten, 1973; Volkov et al., 2011a; Gruzinov, 2011). This is often referred to as blow off resulting in escape rates much larger than the Jeans rate. Blow off has been suggested to occur when $\lambda_x \lesssim 2$ at the exobase altitude or below (Hunten, 1973; Watson et al., 1981).

The slow hydrodynamic escape (SHE) model, considered intermediate to Jeans and hydrodynamic escape regimes, has been suggested to be applicable to a dense tightly bound atmosphere for which the Jeans parameter, $\lambda(r_0)$ is larger than 10, estimated at a radial distance, r_0 , considered to be in approximate thermal and radiative equilibrium (e.g., Parker, 1964b; Watson et al., 1981). The flow is referred to as slow because near r_0 the gravitational energy Φ_g dominates the thermal energy ($C_p T$), which also dominates the flow energy ($1/2 mu^2$) where C_p is the heat capacity per molecule and $u(r)$ is the flow speed. However the flow eventually reaches supersonic speeds above the exobase resulting in escape rates much larger than the Jeans rates. Below we provide a more detailed discussion on the application of hydrodynamic models to the slow hydrodynamic escape regime.

In the 1D steady-state hydrodynamic model the continuity equation leads to a constant molecular flow, given here as a flow rate, ϕ , versus radial distance:

$$\phi = 4\pi r^2 n(r) u(r) = 4\pi r_0^2 n(r_0) u(r_0) \quad (1a)$$

The radial momentum equation, in which the viscous term is dropped, is:

$$\frac{dp}{dr} = n \left(\frac{d\Phi_g}{dr} - \frac{d}{dr} \left(\frac{1}{2} mu^2 \right) \right) \quad (1b)$$

with the gas pressure $p = nkT$. Finally, the corresponding energy equation is:

$$\frac{d}{dr} \left\{ \phi \left(\frac{1}{2} mu^2 + C_p T - \Phi_g \right) - 4\pi r^2 \kappa(T) \frac{dT}{dr} \right\} = 4\pi r^2 Q(r) \quad (1c)$$

where $\kappa(T)$, is the thermal conductivity, C_p is the heat capacity per molecule and $Q(r)$ accounts for the solar heating and IR cooling rates. Knowing the number density, n_0 , and temperature, T_0 , at the lower boundary, i.e. $n(r_0) = n_0$, $T(r_0) = T_0$, Eqs. (1b) and (1c) are solved. A unique solution requires two additional parameters at the lower boundary, u_0 (or ϕ) and $(dT/dr)_0$. Unfortunately, in order to find u_0 and $(dT/dr)_0$, one needs to impose assumptions about the solution behavior at $r \rightarrow \infty$.

Parker (1958) used the hydrodynamic equations to model the thermal expansion of the solar wind in the vicinity of $\lambda(r_0) \sim 2$. He subsequently extended that model to describe the expansion of a stellar wind from a star with a tightly bound corona with $\lambda(r_0) \gtrsim 10$ for which no or very little heat is deposited above r_0 (Parker, 1964a,b). In this formulation, escape is powered by the heat flow from below r_0 and the conditions imposed are $T, n \rightarrow 0$ as $r \rightarrow \infty$. It was then shown that the dense atmosphere must expand according to a critical solution, where the flow velocity, u , gradually increases above the isothermal speed of sound. Purely subsonic solutions were not permitted because they resulted in a finite pressure at infinity.

Chamberlain (1960, 1961) re-considered the expansion of the solar wind for subsonic velocities with the condition $T \rightarrow 0$ as $r \rightarrow \infty$. He deemed this approach to be a slow hydrodynamic expansion of the solar wind, and showed it is possible to obtain a

subsonic solution with the hydrodynamic equations if the energy flux at infinity is 0. In this formulation the number density n approaches a constant as $r \rightarrow \infty$. Later, Parker (1964b) acknowledged this result as a limiting case to supersonic expansion. He determined that this approximation would only occur in the limit that the density at the lower boundary, n_0 , goes to infinity. He showed for sufficiently large densities at r_0 , the energy flux carried to infinity is non-zero for the condition $T \rightarrow 0$ and, hence, the expansion can proceed supersonically. This is the typical approach used in applying the SHE model to planetary atmospheres (e.g., Krasnopolsky, 1999; Strobel, 2008a,b; Watson et al., 1981). Since the flow is slow, the standard procedure is to integrate Eqs. (1b) and (1c) neglecting the u^2 terms (Parker, 1964b). Therefore, although $\varphi \neq 0$ in Eq. (1a), u^2 is set equal to 0 in Eqs. (1b) and (1c) below an upper boundary where $1/2mu^2 \ll C_p T$, and T and n are only regarded as valid out to an r where the u is a small fraction of the local sound speed (McNutt, 1989; Krasnopolsky, 1999; Strobel, 2008a,b).

The SHE model has been subsequently applied to the thermal expansion of planetary atmospheres in which solar EUV and UV heating powers escape above r_0 (Watson et al., 1981). Particular emphasis has been placed on Pluto's atmosphere which is widely thought to be escaping hydro-dynamically (e.g., McNutt, 1989; Krasnopolsky, 1999; Strobel, 2008a; Tian and Toon, 2005). For example McNutt (1989) solved the hydrodynamic equations neglecting the term $C_p T$ in Eq. (1c) to obtain an analytical solution for n , T , and the escape rate φ , assuming the solar heating occurred in a narrow region of the atmosphere. Krasnopolsky (1999) retained the $C_p T$ term, using numerical methods to solve the hydrodynamic equations.

More recently Strobel (2008a,b) applied the SHE model to the atmospheres of Titan and Pluto using more realistic lower boundary conditions at a radial distance where the atmosphere is in approximate radiative equilibrium. He iteratively solved Eqs. (1b) and (1c) using assumed values of φ and $(dT/dr)_0$, to find a solution with the right asymptotic behavior and zero total energy flux at $r \rightarrow \infty$ for Pluto (Strobel, 2008a) and matched to available density data for Titan (Strobel, 2008b). Calculated N_2 escape rates from Pluto, $\varphi \sim 9.4 \times 10^{26} \text{ s}^{-1}$, and Titan, $\varphi \sim 1.5 \times 10^{27} \text{ s}^{-1}$, were several orders of magnitude larger than the Jeans escape rates calculated using the corresponding SHE model exobase densities and temperatures. For example, the most recent SHE model estimate for escape from Pluto's atmosphere is $\sim 10^3$ times the Jeans rate for the suggested atmospheric structure at solar minimum conditions (Strobel, 2008a).

Tucker and Johnson (2009) tested the results for Titan using a kinetic approach and did not obtain large escape rates. In fact, when the temperature in Titan's upper atmosphere was artificially increased so that $\lambda_x \sim 11$, similar to that at Pluto, the escape rate obtained was enhanced over the Jeans rate but only by a factor of ~ 1.5 . Using a kinetic model, Volkov et al. (2011a,b) showed that thermal escape rate from both monatomic and diatomic atmospheres, for which most of the heating occurs below r_0 , differs from the Jeans rate by less than a factor of 2 if $\lambda(r_0) \gtrsim 6$. Such a drastic difference between fluid and kinetic simulations in the domain of the slow hydrodynamic escape, $\lambda(r_0) \gtrsim 10$, is due to the incorrect treatment of the rarefied region of the atmosphere in the hydrodynamic approximation.

The discrepancy between results obtained from solving the hydrodynamic equations and kinetic simulations can be resolved by using a combined fluid and kinetic approach (e.g., Marconi et al., 1996). A stand-alone kinetic simulation is computationally infeasible at $Kn(r_0) \sim 10^{-6}$ characteristic of the density at the lower boundary typically used in modeling escape from Pluto's atmosphere. Therefore, a computationally efficient model can be constructed by coupling the hydrodynamic equations for the dense atmosphere with kinetic simulations for the exosphere region.

3. Combined fluid/kinetic model of thermal escape

A fluid/kinetic model is applied from a lower boundary r_0 in the atmosphere, considered to be in approximate local thermodynamic equilibrium, to a top boundary r_1 where the atmospheric flow is essentially free of collisions. We divide the atmosphere into two regions, a fluid region where the hydrodynamic equations are applicable from r_0 where $Kn \ll 1$ to an intermediate boundary r_{od} chosen to correspond to $Kn \sim 0.1$, and a kinetic region where kinetic simulations are performed by means of the direct simulation Monte Carlo (DSMC) method (Bird, 1994), from r_{od} to r_1 where $Kn \gg 1$.

When solving the fluid equations we make no assumptions about the n and T at infinity. Consistent with the SHE model we drop the $u(r)^2$ terms. It is possible to include such terms in both the SHE and fluid/kinetic approaches. However, for a dense gravitationally bound atmosphere $u(r)^2$ can be safely neglected below the exobase (Parker, 1964b). The lower boundary conditions in the fluid region are n_0 and T_0 , while the parameters φ and $\langle E\varphi \rangle_{r_0}$, the particle and energy flow across r_0 , in Eq. (2b) are determined by the DSMC part of the model. The pressure and the heat flow, given by Eqs. (2a) and (2b), are determined from the integration of Eqs. (1b) and (1c) using the total heating rate $\beta(r) = r_0^{-2} [\int_{r_0}^r r^2 Q(r) dr]$ with $\beta \rightarrow \beta_0$ as $r \rightarrow \infty$, as defined in Strobel (2008a), and $\langle E\varphi \rangle_{r_0}$ is obtained as the constant of integration:

$$p = p_0 \exp \left[- \int \frac{\lambda(r)}{r} dr \right] \quad (2a)$$

$$\varphi(C_p T - \Phi_g) - 4\pi r^2 \kappa(T) \frac{dT}{dr} = \langle E\varphi \rangle_{r_0} + 4\pi r_0^2 \beta(r) \quad (2b)$$

Since the fluid model requires initial values for φ and $\langle E\varphi \rangle_{r_0}$, for the $\beta_0 = 0$ case we began by assuming an isothermal, hydrostatic atmosphere, $d(nkT_0)/dr = n[d\Phi_g/dr]$, and used a DSMC simulation for such an atmosphere starting at $Kn(r_{od}) \sim 0.1$ to obtain the initial estimates. Therefore unlike the SHE model we do not assume that the energy flow at infinity is zero. In a steady state atmosphere energy conservation requires that the energy carried off by escaping molecules is replaced by a flow of energy into the lower boundary, $\langle E\varphi \rangle_{r_0}$, with $\langle E\varphi \rangle = \langle E\varphi \rangle_{r_0} + 4\pi r_0^2 \beta_0$. With such starting conditions, the fluid/kinetic model typically obtained a converged solution in four iterations. That is, the temperatures agree within <3% and densities agree within <2% in the region where $0.1 < Kn < 1$.

As schematically presented in Fig. 1 we solve Eqs. (2a) and (2b) for the density and temperature up to the exobase and iteratively obtain φ and $\langle E\varphi \rangle_{r_0}$. From that solution the resulting n_{od} and T_{od} at a radius r_{od} where $Kn \sim 0.1$, which is about two scale heights below the nominal exobase, are used in the DSMC simulation up to an altitude many scale heights above the exobase, $Kn \gg 1$. The DSMC method tracks a representative sample of atmospheric molecules which are under the gravitational influence of Pluto and subjected to mutual collisions. At the upper boundary of the kinetic domain we obtain the particle escape rate, φ , and the energy flow through the 1D system, $\langle E\varphi \rangle$. Values of $\langle E\varphi \rangle$ and φ from the upper boundary are used to update corresponding values in the fluid part of the model and then used to solve Eqs. (2a) and (2b) up to $Kn = 1$ during the next iteration. Likewise the results from the new simulation of the hydrodynamic equations provide updated n_{od} and T_{od} at $Kn(r_{od}) \sim 0.1$ for the kinetic model, which are then used to obtain new values of φ and $\langle E\varphi \rangle$. This procedure is repeated until we reach consistent densities, temperatures and flow velocities in the region where the fluid and kinetic model overlap. That is, for $Kn \leq 1$ we numerically solve Eqs. (2a) and (2b) to obtain $n(r)$ and $T(r)$. The flow velocity, $u(r)$, is given by Eq. (1a) for the set of escape parameters φ and $\langle E\varphi \rangle$ obtained from the DSMC simulations. We consider a converged solution acceptable when the temperatures

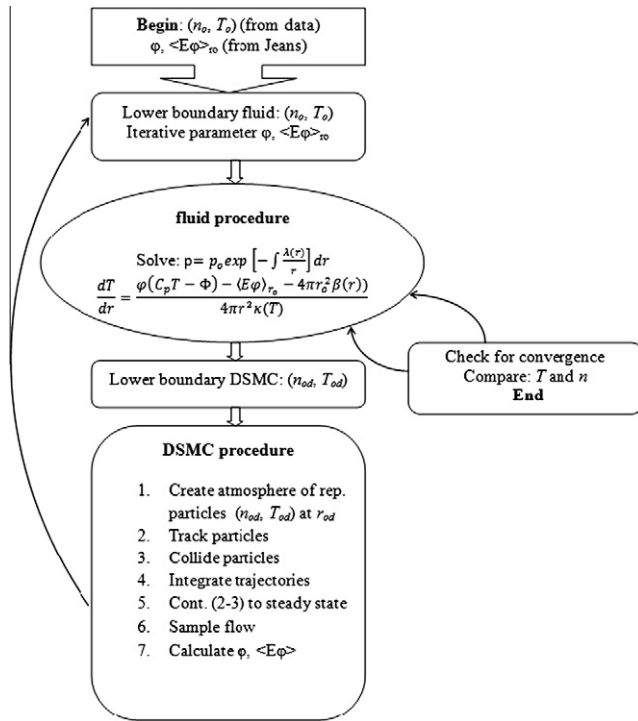


Fig. 1. Schematic of the numerical implementation of the fluid/kinetic model. To obtain solutions of the hydrodynamic equations at r_0 where $Kn < 0.1$, n_0 and T_0 are given, and ϕ and $\langle E\phi \rangle_{r_0}$ are obtained iteratively using the DSMC method. An initial guess of the energy flow into the lower boundary $\langle E\phi \rangle_{r_0}$ is used to solve Eqs. (2a) and (2b) up to $Kn \sim 1$. From the fluid solution n_{0d} and T_{0d} calculated at $Kn_{0d} \sim 0.1$ are used in the DSMC simulation up to $Kn \gg 0.1$ to obtain updated ϕ and $\langle E\phi \rangle_{r_0}$. The iterations are continued until temperature, density and heat flux are consistent between the fluid and DSMC solutions in the overlap region of $0.1 < Kn < 1$.

and densities agree within $\approx 3\%$ in the overlap region between our fluid and kinetic models where $0.1 < Kn < 1$.

In the low Knudsen number regime Eqs. (2a) and (2b) were solved simultaneously using a 4th order Runge–Kutta method with the adaptive radial step-size Runge–Kutta–Fehlberg method (Burden and Faires, 2005), to ensure a relative tolerance of 10^{-8} for $n(r)$ and $T(r)$. The integration steps were between 0.1 and 2 km with the finer resolution necessary to resolve the faster change in temperature near the lower boundary and the narrow heating peak. The heating/cooling models described in Strobel (2008a) are used for the solar heating due to N_2 and CH_4 absorption bands (including UV, EUV, and near-IR) and CO radiative cooling. Since the heating and cooling rates depend on the temperature and column of gas above a given radial position in the atmosphere, the spatial distribution in the net heating rate is recalculated using the new density profile. These iterations are performed until energy conservation is achieved between $\langle E\phi \rangle_{r_0}$, β_0 , and $\langle E\phi \rangle$.

The DSMC method applied in the kinetic region $Kn > 0.1$ in effect solves the Boltzmann kinetic equation by the directly modeling the stochastic nature of the molecular motion in the gas flow using Monte Carlo techniques (Bird, 1994). The DSMC method uses a set of modeling molecules in order to calculate the gas properties of the atmosphere at a molecular level. Collisions between molecules are calculated in discrete radial cells based on the local values of the relative speed, cross section and density. Therefore, the DSMC method is a direct approach for describing the transition in an atmosphere from collisional to collisionless flow. In such a model the conductive heat transfer is represented microscopically.

At the lower boundary of the DSMC domain, r_{0d} , the density n_{0d} and temperature T_{0d} are taken from the solutions of the fluid equations as discussed. Although in the DSMC simulations molecular

motion and collisions are tracked in 3D, in this paper, Pluto's atmosphere is assumed to be spherically symmetric and so the resulting properties depend only on r , consistent with previous models for Pluto's atmosphere (Krasnopolsky, 1999; McNutt, 1989; Strobel, 2008a). Therefore, in the simulation domain when evaluating the collision probabilities the molecular velocities and positions are rotated to a common radial axis. When molecules traverse the DSMC upper boundary, those with velocities greater than the escape velocity and directed outward are assumed to escape and the others are specularly reflected. The reflected molecules represent molecules with trajectories that would eventually return to the simulation domain.

In the DSMC method the time step is chosen to be much smaller than the mean collision time and the cell widths in the flow direction are kept much smaller than l_c and H following the general recommendations in Bird (1994). We use variable cell widths in the radial direction which were approximately 1/3rd of the local mean free path and capped at 10% of the local scale height for mean free paths larger than the local atmospheric e-folding. A time step of ~ 1 – 2.5 s provided energy conservation and ensured that every molecule would have no more than 1 collision over a time step on average. After $\sim 5 \times 10^6$ s the macroscopic properties of the flow were sampled for an additional 5×10^7 s. The number of representative molecules was chosen to ensure a sufficient number of molecules (>200) in the upper most cell, typically we used several 10^3 – 10^5 representative molecules to describe the flow in the kinetic region. The upper boundary location was increased until the escape rate varied by less than 5% with increasing upper boundary. Likewise, when using the converged fluid solution with a DSMC lower boundary deeper in the atmosphere, i.e., choosing a point from the fluid solution between $0.01 < Kn(r_{0d}) < 0.1$, did not significantly affect the results.

Collisions between atmospheric molecules were computed using both the hard sphere (HS) model and the variable hard sphere (VHS) model (Bird, 1994). To ensure consistency between the fluid and kinetic models, we also used the Larsen–Borgnakke (LB) approximation for internal energy and the VHS cross section is parameterized to the temperature dependent thermal conductivity $\kappa(T) = \kappa_0 T^\omega$ for the Maxwell gas, $\omega = 1$. The VHS cross section, relevant for low speed molecular collisions, depends on the relative collision speed v_r , $\sigma = \sigma_0 (\langle v_r \rangle / \langle v_r \rangle)$: where σ_0 is a reference cross section determined from the thermal conductivity and $\langle v_r \rangle$ is the average relative velocity with both values obtained for T_0 assuming the Maxwellian speed distribution. At temperatures characteristic for Pluto's upper atmosphere, the N_2 vibrational modes are assumed not to be excited so that the LB model is used only for two rotational degrees of freedom. The initial internal energy for each molecule is set at the lower boundary of the DSMC regime, r_{0d} , based on a Maxwell–Boltzmann energy distribution and neglecting changes in rotational levels due to IR cooling between collisions.

4. Results for Pluto's atmosphere

The 1D radial fluid/kinetic model is applied to a region in Pluto's atmosphere from $r_0 = 1450$ km up to $r_1 = 30,000$ km for $n_0 = 4 \times 10^{12} \text{ cm}^{-3}$ ($Kn_0 \sim 10^{-6}$) and $T_0 = 88.2$ K consistent with Strobel (2008a). Pluto's orbital axis is nearly parallel to its orbital plane which will result in the structure of the atmosphere being non-isotropic over the globe, and the amount of solar heating is also variable dependent upon the relative abundances of CH_4 and CO present in the atmosphere. However, for the purpose of this study we assume a globally averaged atmosphere and adopted solar minimum heating rates from Strobel (2008a) to compare with the SHE model results. Further studies should be done using a 3D model as the amounts of CH_4 and CO in the atmosphere are better constrained.

We first obtained a converged solution for density and temperature versus radial distance with the fluid/kinetic approach using the HS collision model with the DSMC simulation. To compare the effects of using the HS, HS-LB, VHS and VHS-LB collision models on φ , $n(r)$, $T(r)$ and $u(r)$ within the DSMC model, the following lower boundary conditions were adopted from the converged fluid/kinetic (HS) model $r_{od} = 2836$ km where $\lambda_{od} = 12$, $n_{od} = 2.9 \times 10^7$ cm³ and $T_{od} = 85.5$ K. Using the conductivity for the Maxwell molecules at T_0 (Strobel, 2008a), we obtained a reference value for the HS cross section of $\sigma_0 = 9.0 \times 10^{-15}$ cm², see Table 1 for the model parameters. The mean free paths at the lower boundary for a collision for HS and VHS molecules are $l_c = (\sqrt{2}n\sigma_0)^{-1}$ and $l_c = (\langle v_{th} \rangle / n\sigma_0 \langle v_{r0} \rangle)$ respectively (Bird, 1994), but the values of l_c and $Kn(r_{od})$ for this particular case are similar: ~ 27 km and ~ 0.1 respectively. While the resulting density profiles did not significantly depend on the choice of the collision model for the parameters n_{od} and λ_{od} given above (e.g. for all results $r_x \sim 3900$ km), the resulting temperature profiles and escape rates were slightly different: e.g., the escape rates are 4.4, 5.1, 4.3 and 4.8×10^{25} s⁻¹ for the HS, HS-LB, VHS and VHS-LB models respectively. Above the exobase, as it is seen in Fig. 2a the translational temperature decreases faster than the rotational temperature, and the perpendicular temperature decreases faster than the radial temperature. At distances increasingly above the exobase the atmosphere cools approximately adiabatically as collisions become increasingly infrequent.

In the transition region of the atmosphere there is an altitude where it is most efficient for molecules to acquire escape trajectories. Below this altitude collisions inhibit escape and above there are too few collisions to produce escape trajectories. In the kinetic region we calculated the average number of escaping molecules produced in each radial cell, $d\varphi/dr_{esc}$, by noting the altitude at which molecules that eventually traverse the top of the simulation domain, $r_1 = 30,000$ km, first attained an escape velocity. Molecules that later lose their escape velocity are dropped from the inventory, so the total escape rate is given by $\varphi = \int_{r_0}^{r_1} (d\varphi/dr)_{esc} dr$. The peak in the escape rate production, Fig. 2b, occurs at the same altitude for the HS ($r \sim 4090$ km) and VHS ($r \sim 3680$ km) models with and without the internal degrees of freedom. The difference in the peak altitude is determined by the conductivity which differs between the VHS ($\kappa \propto T$) and HS models ($\kappa \propto T^{1/2}$). For the fluid/kinetic results discussed further below we used the VHS-LB model in order to have both $\kappa(T)$ and C_p consistent with the fluid model, and to allow for rotational/translation energy exchange.

Results are given in Table 2 from two cases obtained using the combined fluid/kinetic simulation with the VHS-LB collision model. In the first case no solar heating occurs in the simulation region: i.e., $Q(r) = 0$ above r_0 so that $\beta_0 = 0$. Next we assume approximate solar minimum conditions where the net heating/cooling above r_0 is such that $\beta_0 = 1.5 \times 10^{-3}$ erg cm⁻² s⁻¹ which is similar to the value used in the SHE model $\beta_0 = 1.7 \times 10^{-3}$ erg cm⁻² s⁻¹ (Strobel, 2008a). The value of β_0 was obtained by using a fixed solar UV and EUV heating efficiency, $\varepsilon \sim 0.25$, with a cut-off in the heating at an altitude where the heat deposited was less than 1% of β_0 (Strobel, 2008a).

Table 1
Parameters for fluid/kinetic model.

Parameter	HS model	VHS model
Heat capacity/molecule: C_p	(3/2) k	(5/2) k
Viscosity exponent: ω ($\kappa(T) = \kappa_0 T^\omega$)	1/2	1 ^a
Collision cross section: σ ($\times 10^{-15}$) cm ²	$\sigma_0 = 9.0$	$\sigma = \sigma_0 (\langle v_{r0} \rangle / \langle v_r \rangle)^b$

Parameters for N₂ used in the fluid/kinetic. The lower boundary radial distance is $r_0 = 1450$ km where $n(r_0) = 4 \times 10^{12}$ cm⁻³ ($Kn_0 \sim 10^{-6}$), $T(r_0) = 88.2$ K.

^a The viscosity exponent for the VHS model and $\kappa_0 = 9.37$ erg cm⁻¹ s⁻² K⁻² are taken from Strobel (2008a).

^b The average relative velocity at r_0 is defined by $\langle v_{r0} \rangle = (16kT_0/\pi m)^{1/2}$.

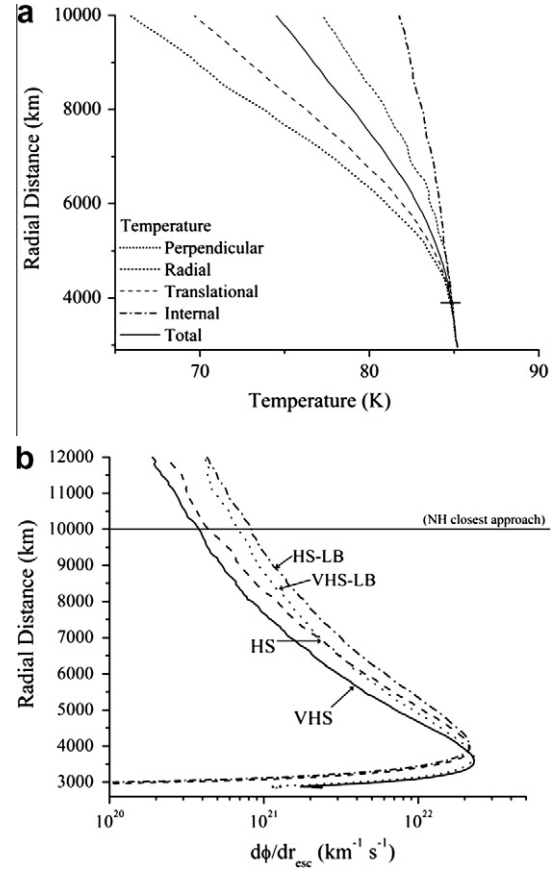


Fig. 2. Results of test simulation described in text $Kn(r_{od}) = 0.1$ and $\beta(r) = 0$ above lower boundary: exobase altitude is 3900 km indicated by short horizontal curve in Fig. 2a: (a) T (K) in VHS-LB model: short dashed curve perpendicular temperature, T_{perp} , dotted curve radial temperature, T_r , dashed dotted curve rotational temperature, T_{rot} , dashed curve translational temperature, $T_{trans} = (T_r + 2T_{perp})/3$, solid curve total temperature, $T = (3T_{trans} + 2T_{rot})/5$. (b) Production of escaping molecules, $d\varphi/dr_{esc}$ (km⁻¹ s⁻¹) versus r : HS (solid curve), VHS (dashed curve), HS-LB (dotted curve), VHS-LB (dashed curve). The New Horizons spacecraft distance of closest approach to Pluto will be 10,000 km.

Table 2
SHE versus fluid/kinetic.

β_0 (10^{-3} erg cm ⁻² s ⁻¹)	No heating		1.7	1.5
	SHE ^a	Fluid/kinetic ^a	SHE ^a	Fluid/kinetic ^a
r_x (km)	2700	3900	3530	6200
n_x ($\times 10^5$ cm ⁻³)	53	17	53	6.7
T_x (K) [H_x (100 km)]	48 [1.2]	85 [4.5]	65 [2.6]	87 [12]
u_x (m s ⁻¹)	1	5×10^{-4}	2	4
λ_x	23	8.8	13	5.4
φ (10^{25} s ⁻¹)	54	4.8	180	120
φ/φ_j	$\sim 10^7$	1.6	$\sim 10^3$	2.0
$\langle E\varphi \rangle / kT_0\varphi$	0	1.8	0	1.8
$\langle E\varphi \rangle_j / kT_0\varphi_j$	~ 1.11	2.02	~ 1.53	2.12

Exobase values for the density, temperature, bulk velocity, escape rate φ and average energy carried off $\langle E\varphi \rangle$ including the corresponding values for the theoretical Jeans escape rate and energy flow rate (φ_j , $\langle E\varphi \rangle_j$) evaluated at the corresponding exobase distances: simulations performed for $\beta_0 = 0$ and solar minimum conditions for the SHE model and the fluid/kinetic (VHS-LB) results shown in Fig. 3. The lower boundary radial distance is $r_0 = 1450$ km where $n(r_0) = 4 \times 10^{12}$ cm⁻³ ($Kn_0 \sim 10^{-6}$), $T(r_0) = 88.2$ K, $\lambda(r_0) = 23$ and $c_0 = 191$ m/s (sound speed).

^a The exobase values for the SHE model are taken from Strobel (2008a).

^a The exobase altitude r_x is determined where $Kn = l_c/H = 1$, $l_c = \langle v_{th} \rangle / (n\sigma_0 \langle v_{r0} \rangle)$.

As seen in Table 2, the escape rate obtained for the $\beta_0 = 0$ case, 4.8×10^{25} s⁻¹, is $\sim 1.6 \times \varphi_j$, where φ_j is the Jeans rate for $T_x = 85$ K and the heat flow out is $\sim 1.4 \times \langle E\varphi \rangle_j$ both evaluated at the

exobase, $r_x = 3900$ km where $Kn = 1$. These results differ significantly from those obtained in the SHE model (e.g., $r_x = 2700$ km, $n_x \sim 5.3 \times 10^6 \text{ cm}^{-3}$, $T_x = 48$ K and $\lambda_x \sim 23$; Strobel, 2008a) indicative of the very different atmospheric profiles as seen in Fig. 3a. The change in temperature with increasing r is seen to fall off much faster in the SHE solution consistent with an overestimate in the adiabatic cooling due to the overestimate in the escape rate. Even though the SHE model has a much larger Jeans parameter at the exobase, the escape rate is an order of magnitude larger than that obtained with the fluid/kinetic model.

Numerically solving the fluid equations for $\beta_0 \neq 0$ is very sensitive to the choice of the input escape parameters, especially the energy carried away by escape, $\langle E\phi \rangle$. Therefore, an initial solution was achieved by incrementally adding in a small fraction of the heating rate and solving the fluid equations iteratively, but assuming Jeans escape at the exobase, $Kn = 1$. Having achieved a converged solution in this manner, we used the calculated $n(r_{od})$ and $T(r_{od})$ evaluated at $Kn(r_{od}) = 0.1$ as the starting point for the first DSMC iteration. When the solar minimum heating is included the Jeans parameter at the lower boundary for the kinetic model for the converged result was $\lambda(r_{od}) \sim 9$. The SHE result taken from Strobel (2008a) is obtained by solving Eqs. (1b) and (1c) using assumed values of ϕ and $(dT/dr)_0$, to find a solution with $n, T \rightarrow 0$ and zero total energy flux as $r \rightarrow \infty$.

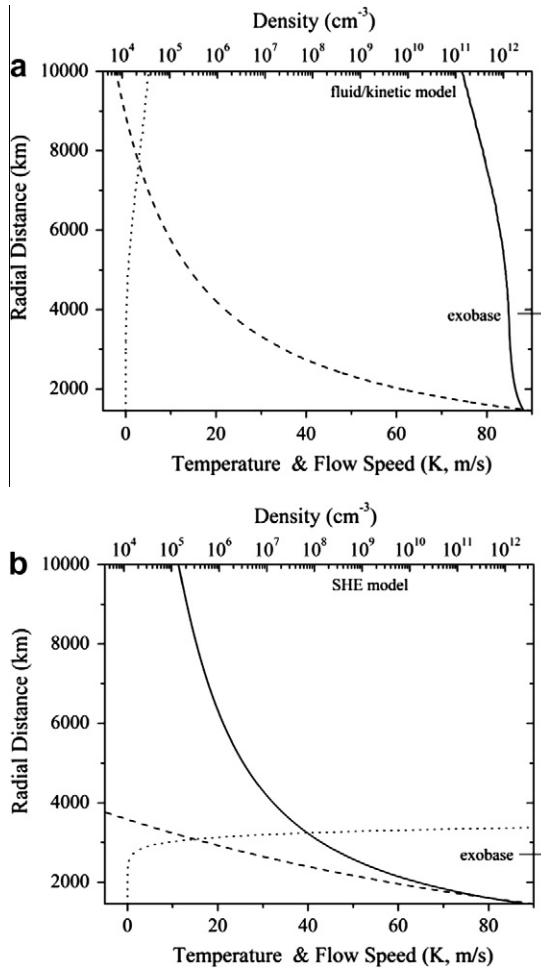


Fig. 3. n (cm^{-3}) (top axis), T (K) and u (m/s) (bottom axis) versus radial distance: Comparison of fluid/kinetic (a) n (dashed curves), T (solid curves) and u (dotted curves) to SHE model results (b) from Strobel (2008a) for no heating above r_0 ($\beta_0 = 0$). The exobase distance is indicated by the solid curve on right axis: 3900 km fluid/kinetic model and 2700 km SHE model. The New Horizons spacecraft distance of closest approach to Pluto will be 10,000 km.

As seen in Table 2, the resulting escape rate for the solar minimum case was $1.2 \times 10^{27} \text{ s}^{-1}$ with $r_x \sim 6200$ km, $n_x \sim 7 \times 10^5 \text{ cm}^{-3}$, $T_x \sim 87$ K and $\lambda_x \sim 5$. Although, the escape rate is fortuitously close to the SHE result, $1.8 \times 10^{27} \text{ s}^{-1}$, the structure of the exobase region for the SHE model is very different: $r_x \sim 3530$ km, $n_x \sim 5.3 \times 10^6 \text{ cm}^{-3}$, $T_x \sim 65$ K and $\lambda_x \sim 13$. Therefore, although the SHE escape rate was suggested to be $>10^3 \times \phi_J$, based on the temperature and density at the exobase obtained here the escape rate is $2.0 \times \phi_J$ and the energy flux rate is $1.7 \times \langle E\phi \rangle_J$. The size of this enhancement to the Jeans rate is similar to that found earlier (Tucker and Johnson, 2009). Ignoring the effect on Charon, this rate is also $\sim 84\%$ of the energy-limited escape rate, $(4\pi r_0^2 \beta_0)/(\lambda_0 k T_0)$, often used in exoplanet studies (Lammer et al., 2009).

5. Effect of Charon on escape

We examine here whether or not these results have implications for Charon, which has 1/2 the diameter and 1/10 mass of Pluto. At an orbital distance of 19,500 km the Hill sphere radius about Charon is at a radial distance of 12,700 km from Pluto, where the atmospheric density is $\sim 10^5 \text{ cm}^{-3}$ for the solar minimum case, Fig. 4a. Ignoring here any tidal effect on Pluto's lower atmosphere we estimated whether the effect of Charon's gravity on the molecular trajectories

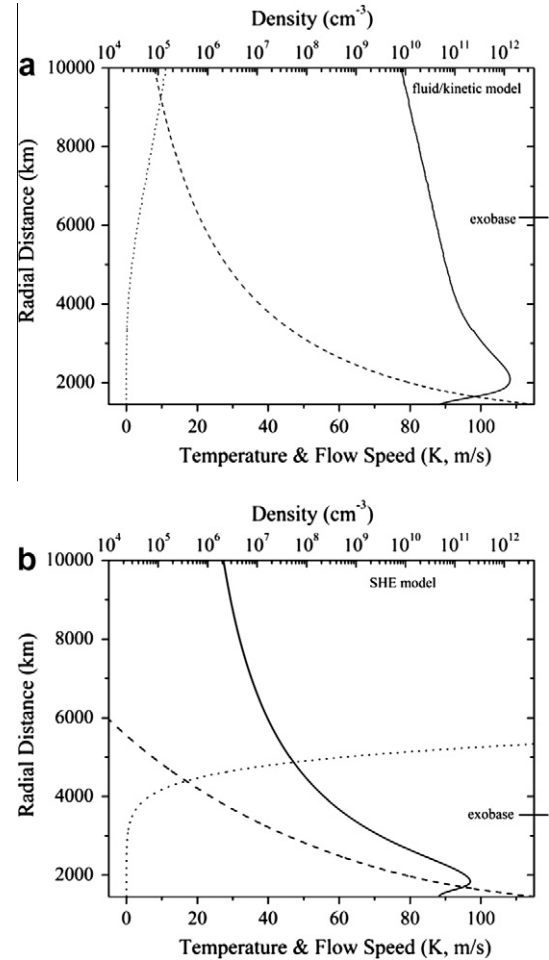


Fig. 4. n (cm^{-3}) (top axis), T (K) and u (m/s) (bottom axis) versus radial distance: Comparison of fluid/kinetic (a) n (dashed curves), T (solid curves) and u (dotted curves) to SHE model results (b) from Strobel (2008a) for solar minimum heating conditions above r_0 . The exobase distance is indicated by the solid curve on right axis: 6200 km fluid/kinetic model and 3530 km SHE model. The New Horizons spacecraft distance of closest approach to Pluto will be 10,000 km.

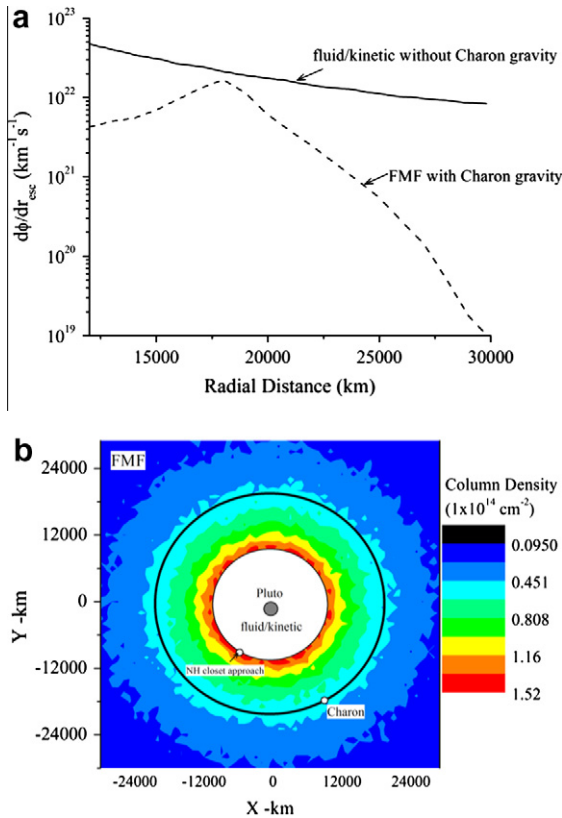


Fig. 5. (a) Production of escaping molecules $d\phi/dr_{\text{esc}}$ ($\text{km}^{-1} \text{s}^{-1}$) versus r . DSMC results for solar minimum heating conditions without including Charon's gravitational influence (solid curve), as in Fig. 2b, compared to the fluid/kinetic result coupled to a FMF model that includes Charon's gravitational effect (dashed curve). (b) Contour plot of column density in the z plane Charon's northern and southern hemispheres in FMF simulations including Charon's gravitational influence: black curve represents Charon's orbit. Particles are emitted radially with corresponding speeds according to the local distribution at 10,000 km. The New Horizons (NH) spacecraft distance of closest approach to Pluto will be 10,000 km.

would significantly affect the escape rate. Therefore, we used the DSMC results for solar minimum conditions to perform free molecular flow (FMF) simulations in which molecules move under the influence of gravity from both Pluto and Charon but without intermolecular collisions. Charon is assumed to have a circular orbit about Pluto and the molecular trajectories are tracked in a 3D region from 10,000 km to 30,000 km about Pluto. The FMF simulation is begun by emitting molecules at $r = 10,000$ km with radial velocities obtained from the fluid/kinetic model for solar minimum heating conditions. Escaping molecules are either emitted initially with a speed above the escape speed or they gain an escape speed under the gravitational influence of Charon. After several Charon orbits we obtained a steady-state morphology of the gas density in the Pluto–Charon system and the integrated escape rate produced versus radial distance, as shown in Fig. 5a and b.

For solar minimum heating conditions we estimated that molecules from Pluto's atmosphere impinge upon Charon at a rate of 10^{25} s^{-1} . With Charon's surface temperature of $\sim 50 \text{ K}$ this would be equivalent to the deposition of a monolayer of molecules over 4 Charon orbits or $8 \times 10^{-3} \mu\text{m/yr}$. We also found that Charon has only a small effect on the escape rate from Pluto's atmosphere. Charon does not trap many molecules from Pluto's expanded atmosphere, but rather perturbs the molecules trajectories accelerating them to or decelerating them from escaping the system. The above conclusions were determined by performing FMF simulations with and without the gravitational influence of Charon in which we found that the escape rate decreased by 3% in simu-

tions that included Charon. This is opposite to the change in the escape rate that would have occurred had we used the energy to reach the Hill sphere of Charon as the escape criterion in the fluid/kinetic simulations. A contour plot of the averaged total column densities over the north and south hemispheres when including Charon's gravitational influence is shown in Fig. 5b. The relevance of Charon is likely more significant at solar maximum heating conditions and close to perihelion.

6. Conclusions

Hydrodynamic models have often been applied to atmospheres in the Solar System and to exoplanet atmospheres in order to estimate escape and the concomitant adiabatic cooling of the upper atmosphere (e.g., McNutt, 1989; Krasnopolsky, 1999; Tian and Toon, 2005; Strobel, 2008a,b; Yelle, 2004; Tian, 2009; Murray-Clay et al., 2009). Unless the Jeans parameter is <2 well below the exobase ($Kn \ll 1$), this procedure can give incorrect atmospheric properties as compared to the fluid/kinetic combined approach described here. The difficulties with using a continuum model of thermal escape are twofold; how to, without prior knowledge, define density, temperature and energy flow at infinity, and how to properly define thermal conduction in the exosphere. The Fourier heat flux used to solve Eq. (2b) becomes invalid near the exobase as discussed earlier (e.g., Johnson, 2010; Volkov et al., 2011a,b). Here we use a combined fluid/kinetic model that explicitly incorporates how heat conduction powers escape without requiring any assumptions about the macroscopic properties of the atmosphere at infinity. The hydrodynamic equations are solved below the exobase, and the kinetic model is continued above where the flow is essentially non-equilibrium. Such a procedure is relevant not only to Pluto but to the evolution of atmospheres on terrestrial bodies including recently discovered hot, rocky exoplanet atmospheres.

For over a few decades, hydrodynamic models have been used to conclude that Pluto's atmosphere is lost by a process called slow hydrodynamic escape (e.g., Krasnopolsky, 1999; McNutt, 1989; Strobel, 2008a,b). We reconsidered escape from Pluto using the fluid/kinetic model and found that for the two cases considered, thermally-driven escape occurs at a rate within a factor of two of the Jeans rate for the temperature determined in the combined model. That is, for a lower boundary, r_0 , in Pluto's atmosphere where $\lambda(r_0) \sim 23$ and $Kn(r_0) \sim 10^{-6}$, and with all of the heat deposited below r_0 (i.e., $\beta_0 = 0$), we obtain an escape rate $\phi \sim 4.8 \times 10^{25} \text{ N}_2 \text{ s}^{-1}$. For the derived exobase temperature, $T_x = 85 \text{ K}$, this is ~ 1.6 times the Jeans rate ($\phi_J \sim 3.0 \times 10^{25} \text{ N}_2 \text{ s}^{-1}$) and 1.4 times the Jeans energy flux ($\langle E\phi \rangle_J \sim 7.40 \times 10^{11} \text{ ergs s}^{-1}$). Furthermore we find that each escaping molecule carries off an energy $\sim 2kT_0$ as seen in Table 2, and not 0 as assumed in the SHE model. It is interesting to note that for the same lower boundary conditions, if one assumed the atmosphere was hydrostatic, the Jeans rate would be $\phi_J \sim 5.6 \times 10^{25} \text{ N}_2 \text{ s}^{-1}$ and $\langle E\phi \rangle_J \sim 1.4 \times 10^{12} \text{ ergs s}^{-1}$. Therefore, these simulations indicate escape is similar in nature to Jeans escape, but to get the correct exobase temperature and density needed to make a Jeans estimate, a kinetic model should be applied in the non-equilibrium region of the atmosphere.

We also simulated Pluto's atmosphere for solar minimum conditions above r_0 . For the same density and temperature at r_0 , with a similar solar minimum heating rate to that used in Strobel (2008a), we obtain $\phi \sim 1.2 \times 10^{27} \text{ N}_2 \text{ s}^{-1}$. At the derived $T_x = 87 \text{ K}$ from the model this is ~ 2.0 times the Jeans rate ($\phi_J \sim 6.0 \times 10^{26} \text{ N}_2 \text{ s}^{-1}$) and ~ 1.7 times the Jeans energy flux ($\langle E\phi \rangle_J \sim 1.6 \times 10^{13} \text{ ergs s}^{-1}$). As seen in Table 2, the escape rate for solar minimum conditions is fortuitously close to that obtained in Strobel (2008a) of $1.8 \times 10^{27} \text{ N}_2 \text{ s}^{-1}$, but the total energy flux into the lower boundary leads to a very different atmospheric structure in the exobase

region. Although this energy flux is a small fraction of the energy flux added above r_0 due to solar heating, $\langle E\varphi \rangle_{r_0}/(4\pi r_0^2 \beta_0) = 6.8 \times 10^{-2}$, it influences density and temperature gradients below the heating peak where $\beta(r) \rightarrow 0$ as $r \rightarrow r_0$. Furthermore the flow fields are radically different. This may be illustrated by comparing the relative magnitudes of the static, p , and dynamic, $1/2\mu u^2$, pressures of each model. For the radial distance examined, 1450–10,000 km, we find the total pressure profile of the fluid/kinetic model monotonically decreases and is dominated by the static pressure. On the other hand, the SHE model experiences a minimum in the dynamic pressure at 5700 km with the region below dominated by the static pressure and the region above dominated by the dynamic pressure. The sum of the dynamic and static pressures in the SHE model also exceeds that of the fluid/kinetic model above 8200 km, suggesting that despite their similar flux rates the two models may be observationally distinguishable in their determination of Pluto's interaction with the solar wind.

We show here that starting at a small $Kn(r_0)$ in Pluto's atmosphere, a combined fluid/kinetic model can lead to reliable energy and molecule escape rates both for no heating and solar minimum heating conditions in the region above 1450 km. It is also clear from the fluid and DSMC results in the overlap region, that for this range of Jeans parameters a fluid model *can* obtain accurate temperatures densities and gas velocities with similar heat fluxes up to the exobase. But this is the case *only* if the φ and $\langle E\varphi \rangle_{r_0}$ used are equal to that obtained from a kinetic simulation of the exobase region. In fact, the total energy flux through the system cannot be determined independently for finite $Kn(r_0)$ using a fluid calculation, because it depends on the flow in the non-equilibrium region of the exosphere. Numerical methods have been used to solve the hydrodynamic model using a Jeans type escape and energy flux at or near the exobase (Chamberlain, 1961; Yelle, 2004; Gruzinov, 2011). However, these models require assumed values for n and T at the upper boundary.

We have treated Pluto's atmosphere using a single species, N_2 , throughout the simulation region and have found an enhanced Jeans rate like that found earlier (Tucker and Johnson, 2009). Although minor species, with very different masses, will separate from the N_2 profile in the region of escape (e.g., Tucker and Johnson, 2009), CO should roughly track the N_2 profile described here. Since the solar activity during the observations of Greaves et al. (2011) in 2009/2010 was close to that used for our assumed solar minimum conditions, the discovery of CO at altitudes ~ 4500 km might not be surprising based on the atmospheric structure found here and does not require understanding the interaction of the extended atmosphere with the solar wind. Based on an assumed mixing ratio of $\sim 0.05\%$ the CO tangential column density at 4500 km would be $\sim 6 \times 10^{11}$ CO cm $^{-2}$, but the temperature at this altitude is 91 K as opposed to 50 K suggested by the observations.

Solar maximum conditions are expected to occur in 2013, so that the New Horizon encounter with Pluto and Charon in 2015 will occur somewhere between solar maximum and minimum conditions. At a distance from the Sun of 33 AU and assuming the same heating efficiency and cooling process, this results in $\beta_0 \sim 1.7$ times that used here ($\sim 2.5 \times 10^{-3}$ erg cm $^{-2}$ s $^{-1}$). Therefore, accurate simulations of the atmospheric density at the encounter distance 10,000 km from Pluto, and the atmospheric structure and the escape rates expected during the encounter will require the use of a fluid/kinetic model such as that described here. Such calculations are in progress for a multispecies atmosphere.

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